International Association of
Computational Mechanics

Second
World Congress
on Computational Mechanics

August 27-31, 1990, Stuttgart, FRG

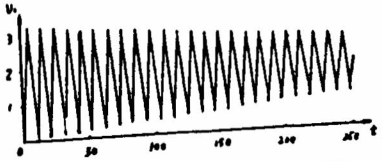

Fig.3 Δt=0.01, U_a Curve by WΘM and ES-WRM(c,s)

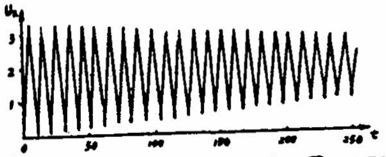

Fig.4 Δt=0.05, U_a curve by ES-WRM(c,s)

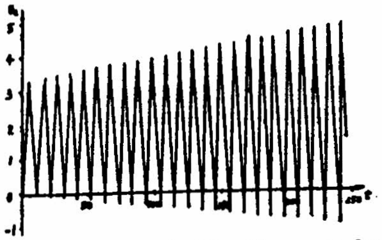

Fig.5 Δt=0.05, U_a curve by WΘM

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NONLINEAR FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE STRUCTURES SUBJECTED TO BLAST AND IMPULSIVE LOADING

by

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1 INTRODUCTION

The ability of reinforced concrete to absorb energy under dynamic loading conditions has led to its utilization for structures which may be subject to impulsive and blast loads. This paper presents a numerical procedure for predicting the non-linear structural response of impulsively loaded 2D reinforced concrete structures. Main attention has been paid to the development of appropriate rate and history dependent material models of concrete and steel. The blast loading modelling has been described in detail elsewhere [1].

2 MATERIAL MODELLING OF CONCRETE BEHAVIOUR

The compressive behaviour of concrete has been modelled as a strain rate sensitive elasto-viscoplastic material. The viscoplastic behaviour is controlled by two strain rate dependent surfaces in the stress space, the initial yield surface F_o and a failure surface F_f. The failure surface predicts failure if the state of stress satisfies the following condition

$$F_f\left(I_1,\ J_2,\ \sigma'_{cd}\right) = f\left(I_1,\ J_2\right) - \sigma'_{cd} = 0 \qquad (1)$$

where $f\left(I_1, J_2\right)$ is the failure function which is assumed to be a function of the first stress invariant I_1 and the second deviatoric stress invariant J_2 and σ'_{cd} is the dynamic compressive strength of concrete. Based on Kupfer's results [2], two different functions are developed for the representation of the failure surface of concrete in the principal stress space such that

$$f_1\left(I_1,\ J_2\right) = a\ I_1 + \sqrt{b\ I_1^2 + 3\ c\ J_2} \qquad (2)$$

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$$f_2\left(I_1,\ J_2\right) = a\ I_1 + \sqrt{a\ \sigma'_{cd} + 3\ c\ J_2} \qquad (3)$$

where a, b, c are material constants which have been evaluated elsewhere [1]. The initial yield surface defines the onset of the viscoplastic behaviour. When the stress state lies within this surface, concrete is assumed to be linear elastic. Once concrete is stressed beyond the elastic limit, a subsequent new loading surface is developed. The loading surface is assumed to have the same shape in stress space as that of the failure surface. Thus, the general form of the loading surface is proposed as

$$F_o\left(I_1,\ J_2,,\ \tau\right) = f\left(I_1,\ J_2\right) - \tau = 0$$

in which τ is the effective stress which is assumed to be a function of the effective viscoplastic strain ε_{vp}, and the effective strain rate, $\dot{\varepsilon}_{eff}$. Due to strain hardening in the pre-fracture range, the loading surface expands with increasing viscoplastic strain. A strain rate sensitive isotropic hardening rule is developed in which the amount of expansion is history and rate dependent. Upon mathematical derivations [1], the normalized effective stress is found as

$$\frac{\tau}{\sigma'_{cd}} = -\frac{2}{c}\left[1 - C + \frac{2C}{\varepsilon'_{cd}}\varepsilon_{vp}\right]^{1/2} - \frac{2}{c}\left[1 - C + \frac{C}{\varepsilon'_{cd}}\varepsilon_{vp}\right] \qquad (5)$$

in which ε'_{cd} is the dynamic peak compressive strain and C is a constant given by

$$C = \frac{4(1-\alpha)}{(2-\alpha)^2} \qquad (6)$$

Excellent agreement [1] has been obtained with Kupfer's results [2]. In the postfailure range, the loading surface shrinks with the increase in viscoplastic strain. A rate dependent softening rule has been defined [1] in which the effective stress is assumed as a function of the dynamic compressive strength and the postfailure viscoplastic energy, K, such that

$$\tau = \sigma'_{cd}\ e^{-\beta\kappa} \qquad (7)$$

in which β is a concrete softening constant. The crushing conditions have been developed by converting the failure functions described in terms of stresses to that in terms of strains [1]. To evaluate the viscoplastic strain rate vector, $\dot{\varepsilon}_{vp}$, the classic associated flow rule is modified to include the rate dependence in which the fluidity parameter γ, is derived as a function of the effective strain rate

$$\dot{\varepsilon}_{vp} = \gamma\left(\dot{\varepsilon}_{eff}\right) < \phi\ (F_o) > a \qquad (8)$$

where $\phi\ (F_o)$ is the flow function and a is the flow vector. Based on the governing uniaxial elasto-viscoplastic stress strain relation, derived in [1], and the curve fitting of the experimental results, the fluidity parameter is found as

$$\gamma = 10^{B_1}\left(\dot{\varepsilon}_{eff}\right)^{B_2} \qquad (9)$$

where B_1 and B_2 are material parameters.

In tension, concrete is modelled as a linear elastic strain softening material where crack initiation is controlled by a rate dependent strain criterion such that,

$$\varepsilon_{cr} \geq \varepsilon'_{td} \qquad (10)$$

where ε'_{td} is the dynamic cracking strain of concrete. The smeared crack approach is employed to simulate cracks. Post-cracking behaviour is governed by an objective non-linear softening rule [3] based on the concrete fracture energy G_f, the crack characteristic length L_c, the static tensile strength σ'_{ts} and static cracking strain ε'_{ts}

$$\sigma = \sigma'_{ts}\ e^{\frac{-\varepsilon + \varepsilon'_{ts}}{\psi}} \qquad (11)$$

$$\psi = \left(G_f - 0.5\ \sigma'_{ts}\ \varepsilon'_{ts}\ L_c\right)\ /\ \sigma'_{ts}\ L_c \qquad (12)$$

The shear transfer across the cracks due to aggregate interlock and dowel action is considered by employing a reduced shear modulus in which the reduction factor is assumed to be a function of the current tensile strain [1].

An important consideration in the present concrete model is the strain rate anistropy introduced by different strain rate sensitivity functions for tension and compression. For concrete in compression, functions available in the literature have been used. A strain rate sensitivity function for concrete cracking strain has been developed [1] by curve fitting of testing data.

3 MATERIAL MODELLING OF REINFORCING STEEL BEHAVIOUR

Steel is modelled as a uniaxial strain rate dependent elasto-viscoplastic material in tension and compression in which the yield stress σ_{yd} is rate dependent. Beyond yield point, the effective stress level is governed by a linear strain hardening function such that

$$\tau = \sigma_{yd} + H\ \varepsilon_{vp} \qquad (13)$$

where H is the hardening modulus of steel. To calculate the viscoplastic strain rate, the associated flow rule is modified in a similar fashion to concrete where the fluidity parameter is derived as a function of the strain rate [1].

4 FINITE ELEMENT FORMULATION AND SOLUTION TECHNIQUE

The semi-discrete form of the non-linear dynamic equilibrium equations can be presented as

$$\mathbf{M}\,\ddot{\mathbf{d}} + \mathbf{C}\,\dot{\mathbf{d}} + \mathbf{R}\,(\mathbf{d}) = \mathbf{Q} \tag{14}$$

where $\mathbf{d}$, $\dot{\mathbf{d}}$ and $\ddot{\mathbf{d}}$ are vectors of nodal displacements, velocities and accelerations respectively. $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, $\mathbf{R}(\mathbf{d})$ is the vector of internal resisting forces and $\mathbf{Q}$ is the vector of external applied blast loads. 8-noded isoparametric elements have been employed for the spatial discretization of equation (14), with embedded bars to simulate reinforcement. Perfect bond is assumed between steel and concrete. Hinton's lumping scheme has been adopted to generate lumped mass matrix from the consistent mass matrix.

Explicit central difference scheme has been employed for the time discretization of equation (14), while explicit Euler integration scheme has been adopted for equations which govern viscoplastic straining. Numerical stability has been controlled using appropriate time steps. However, due to material non-linearity, an additional energy balance check becomes necessary to guard against possible errors generated through numerical instability [1].

5 COMPUTER IMPLEMENTATION AND NUMERICAL APPLICATION

A versatile and comprehensive computer program, FEABRS, has been developed for the finite element linear and non-liner dynamic analysis of 2D reinforced concrete structures subjected to impulsive and blast loading. The program has been written in modular form, embodies the material models described above, and possesses sufficient flexibility to add new options resulting from further research. The program has been used to study the response characteristics of reinforced concrete beam under impulsive loading, shown in Figure 1. Figure 2 shows the time history of the midspan deflection which is found to be in a very good agreement with that given in [4]. The paper will include the results of parametric study to establish the generality of the proposed computational models.

6 PRINCIPAL CONCLUSIONS

1. The finite element method in conjunction with suitable material modelling is a good tool for predicting the behaviour of concrete structures subjected to impulsive loading.

2. The modified theory of viscoplasticity which includes the strain rate effects and strain hardening and softening is suitable for simulating the dynamic compression behaviour of concrete. However, tensile cracking plays a dominant role in the non-linear behaviour of concrete.

3. The central difference scheme in conjunction with energy balance check for numerical integration of non-linear equations of motion proves to be efficient in tackling complex material models.

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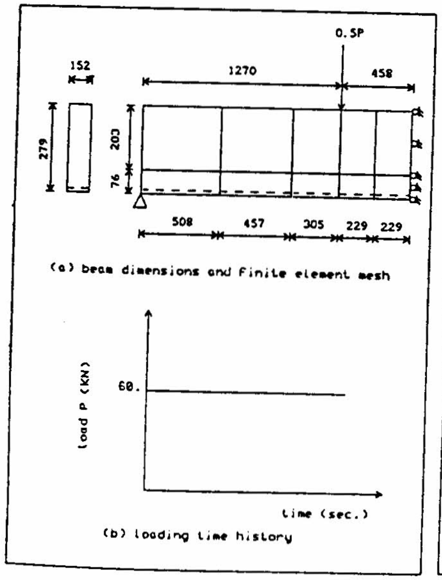

Figure 1 Geometry, mesh and loading for reinforced concrete beams [4]

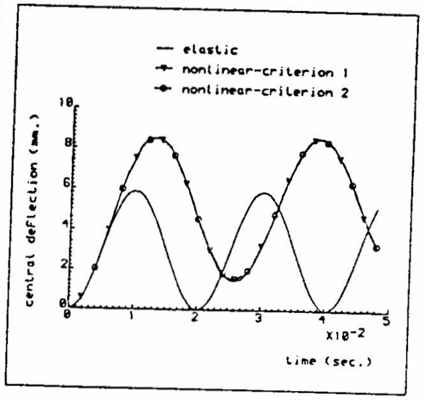

Figure 2 Linear and non-linear response of the RC beam